

Mathematical Models for Energy Simulation in Livestock Systems: A State of the Art

Modelos Matemáticos para la simulación energética en sistemas ganaderos: un estado del arte

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ABSTRACT: Bovine production systems represent one of the most significant productive activities in Latin America due to their contribution to food security and their role in the rural peasant economy. However, their progressive development in recent years has raised concerns regarding environmental, economic, and social sustainability-factors that have not evolved favorably alongside such growth. In this context, analyzing energy flows and the adaptation of natural resources within bovine production systems becomes a critical need to advance toward more efficient and sustainable models. This article presents a systematic review of the scientific literature on the application of mathematical, energetic, and emergy-based models in bovine production systems. It explores classical approaches to animal growth, multivariate simulation tools and software, and emergy analysis methods using indicators such as EYR (Emergy Yield Ratio), ELR (Environmental Loading Ratio), and ESI (Emergy Sustainability Index). Additionally, it examines notable case studies from Latin America and Europe, highlighting methodological strengths, limitations, and opportunities for integration. As a result, five strategic lines are proposed for the development of context-specific hybrid modeling frameworks.

Keywords: Energy Efficiency, Food Security, Rural Economy, Agriculture Ecosystems.

RESUMEN: Los sistemas de producción bovina hacen parte de una de las actividades productivas más relevantes en América Latina, debido a su aporte a la seguridad alimentaria y el papel que tiene en la economía rural campesina. Sin embargo, su desarrollo progresivo en los últimos años ha generado cuestionamientos en torno a la sostenibilidad ambiental, económica y social, factores no favorables por dicho progreso. En este sentido, el análisis del flujo de energía y la adaptación de los recursos naturales en los sistemas de producción bovina se convierte en una necesidad crítica para avanzar hacia modelos más eficientes. Este artículo presenta una revisión sistemática de la literatura científica sobre la aplicación de modelos matemáticos, energéticos y emergéticos en sistemas de producción bovina. Explorando enfoques clásicos de crecimiento animal, herramientas y software de simulación multivariada, y métodos emergéticos basados en indicadores como el EYR, ELR y ESI. Asimismo, se analizan algunos estudios de caso sobresalientes en América Latina y Europa, destacando fortalezas, limitaciones y posibilidades de integración metodológica. Por lo tanto, se proponen cinco líneas estratégicas para el desarrollo de modelos híbridos contextualizados.

Palabras clave: eficiencia energética, seguridad alimentaria, economía rural agroecosistemas.

INTRODUCTION

Among the tools that address these challenges, emergy analysis stands out as a systemic approach proposed by Odum (1996), which evaluates the energy accumulated-both directly and indirectly-in the generation of goods and services within a system, expressed in solar equivalent joules (sej). Unlike conventional energy metrics,

emergy integrates hidden environmental costs and allows for the assessment of sustainability based on the overall ecological performance of the system (Vigne *et al.*, 2013; Rotolo *et al.*, 2007). In livestock systems, this approach has been used to compare extensive and intensive models, estimate the ecological support required by production practices, and establish environmental efficiency indicators such as EYR, ELR, and ESI.

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In parallel, advancements in computational sciences and quantitative methods have facilitated the development of mathematical models capable of simulating the energetic behavior of livestock systems under different management, climate, and demand scenarios (Tedeschi, 2004; Gómez et al., 2006). Models such as Brody, Gompertz, Richards, and Von Bertalanffy, traditionally employed to describe animal growth, have been adapted to predict energy intake, metabolic flows, and conversion efficiency in beef and dairy cattle. Additionally, multivariate techniques such as principal component analysis (PCA) and cluster classification have been instrumental in characterizing production typologies, as demonstrated in recent studies in Colombia (Cruz et al., 2023; Durana et al., 2023).

Despite these advances, there remains a significant gap in the literature regarding the integration of mathematical models with emergy analysis in livestock systems. Research often focuses on either productive or energetic aspects in isolation, lacking a holistic framework to comprehend the biophysical, socioeconomic, and ecological complexity of bovine systems (Postigo et al., 2024; Molina et al., 2025). This gap limits the formulation of sustainable intensification strategies, particularly in rural and small-scale contexts where production decisions must account for both ecological constraints and local knowledge.

Therefore, the objective of this article is to provide an updated review of the state of the art on mathematical models applied to energetic and emergy simulation in bovine livestock systems, with a particular focus on their use as tools for diagnosing and redesigning more sustainable production systems. The review explores both classical and contemporary models, their applicability across various agroecological contexts, the metrics and indicators employed, and emerging trends in modeling bovine agroecosystems from a systemic perspective.

DEVELOPMENT OF THE TOPIC

Foundations of Simulation and Modeling in Livestock Agroecosystems

The analysis of livestock agroecosystems, particularly beef and dairy cattle systems, requires methodological approaches capable of capturing their biophysical, ecological, and socioeconomic complexity. These systems operate as open and interdependent units, in which climatic, edaphic, biological, technological, and human factors interact, affecting both productive efficiency and energy sustainability (Tedeschi, 2004; Durana et al., 2023). For this reason, the use of mathematical models and simulations has become highly relevant to represent, analyze, and project the dynamic behavior of these systems under different scenarios.

Systemic Approach and the Dynamics of Livestock Systems

The systemic approach, initially proposed by Odum (1996), makes it possible to identify both internal processes (growth, reproduction, feed conversion into biomass) and external ones (solar energy input,

external inputs, ecosystem services), which represent the flows of matter and energy within the agroecosystem of bovine production systems. From this perspective, models not only quantify productive outputs but also account for hidden costs and the system's dependencies on its biophysical environment.

This approach has proven useful for comparing contrasting management systems (extensive vs. intensive), identifying critical points of energy loss, and assessing the ecological support required to sustain production (Vigne et al., 2013; Cruz et al., 2023). Moreover, it enables the integration of socio-environmental elements such as climate resilience, land use, and territorial sustainability (Postigo et al., 2024).

Types of Mathematical Models Applied in Livestock Production

There are multiple types of models used to simulate processes in livestock systems, which can be classified according to their structure and purpose into:

Deterministic models, such as those of Brody, Gompertz, Richards, or Von Bertalanffy, describe animal growth using mathematical functions fitted to weight, age, or intake data. These models have been applied to both beef and dairy breeds and allow for the prediction of productive performance under standard conditions (Gómez et al., 2008).

Stochastic and probabilistic models incorporate uncertainty and variability in input parameters, such as Monte Carlo or Bayesian models. These are useful for simulating risk scenarios, changes in feed availability, or climatic variability (Tedeschi, 2004).

Multivariate models include principal component analysis (PCA), cluster analysis, and dimensionality reduction techniques. These tools enable the classification of livestock farms based on climate, altitude, technology adoption, productivity, and energy efficiency, as demonstrated in studies conducted in Cundinamarca and the Andean highlands (Cruz et al., 2023; Molina et al., 2025).

System dynamics simulation models, such as those developed with EmSim, use differential equations and energy diagrams (based on Odum (1996) framework) to model emergy flows, resource accumulation, ecological efficiency, and the transformation of livestock products (Rotolo et al., 2007).

Energetic and Emergy-Based Modeling

Traditional energy modeling in bovine livestock production has focused on estimating the consumption and use of energy in the form of feed, human labor, fossil fuels, machinery, and inputs. These models enable the calculation of net energy balance, conversion efficiency, and energy productivity, and have been fundamental in determining the technical feasibility of different production schemes (Durana et al., 2023; Cruz et al., 2023).

However, this perspective often overlooks hidden environmental costs, and the ecosystem services used indirectly. This is where emergy-based modeling offers a more holistic perspective, by considering not only the quantity of energy but also its quality, origin, and level of transformation. The emergy approach, developed by Odum (1996), allows for the representation of accumulated flows of solar equivalent energy (seJ) required to sustain both ecological and productive processes in a livestock system.

Based on these flows, emergy indicators are constructed and have been widely validated in agricultural sustainability studies (Rotolo *et al.*, 2007; Vigne *et al.*, 2013). Among the most relevant indicators are:

Emergy Yield Ratio (EYR): measures how much emergy is generated per unit of emergy invested from outside the system. High values indicate greater self-sufficiency or dependence on local resources (such as native forage and rainfall). *Example:* An extensive system with natural forage and minimal mechanization may have an EYR > 8, whereas an intensive system with heavy use of concentrate feed may have an EYR < 2.

Environmental Loading Ratio (ELR): represents environmental pressure by relating non-renewable emergy and human services to renewable emergy. Lower values indicate lower environmental impact. *Example:* An intensive system with synthetic fertilization, fossil fuel use, and machinery may have an ELR > 3, suggesting a high dependency on external resources.

Emergy Sustainability Index (ESI): the ratio of EYR to ELR, integrating performance and environmental burden into a single indicator. An ESI > 5 indicates a sustainable system; an ESI < 1 suggests unsustainability. *Example:* Pasture-based systems with agroecological management may have ESI > 20, whereas industrialized systems may fall below 0.5.

RESULTS AND DISCUSSION

Starting from the stated objective regarding mathematical models applied to the energetic and emergy simulation of bovine livestock systems-with particular emphasis on their use as tools for diagnosing and redesigning more sustainable production systems-this section reviews both classical and contemporary models, their applicability across various agroecological contexts, the metrics and indicators employed, and emerging trends in the modeling of bovine agroecosystems from a systemic perspective.

Review of the State of the Art on Energetic and Emergy-Based Modeling in Bovine Livestock

Over the past two decades, scientific literature has shown a growing interest in the use of mathematical models and energy flow analyses to assess the performance and sustainability of Bovine livestock systems. This review is organized around three core themes: (i) studies applying animal growth models for energy-related purposes;

(ii) research modeling energy or emergy flows in production systems; (iii) studies integrating multivariate analyses or advanced simulations to classify or redesign livestock systems, as well as the current gaps and opportunities in the modeling of bovine production systems.

Application of Animal Growth Models for Energetic Purposes (Extended Version)

Animal growth models have traditionally been used to describe the biological development of livestock, but their application in energy analysis has enabled simulations of feed intake, feed efficiency, and energy requirement predictions (Perdigón & González, 2020). Among the most widely used models are Brody, Gompertz, Richards, and Von Bertalanffy.

By being fitted to real productive data, these models allow for the estimation of energy requirements for maintaining specific growth rates, projecting feed conversion efficiency, and improving nutritional planning (Perdigón & González, 2020). Recent integrations with programming languages such as R and Python, or simulation platforms like Stella, have enhanced their utility.

Energetic and Emergy-Based Modeling in Livestock Systems

Conventional energy modeling in livestock has focused on estimating the relationship between inputs and outputs within production systems. Inputs include feed (forage, concentrates), fossil energy (fuels, electricity), labor, and pharmaceuticals, while outputs include meat, milk, manure, and other derived products (Durana *et al.*, 2023; Cruz *et al.*, 2023). These evaluations make it possible to build energy balances and analyze conversion efficiency, yet they often overlook the quality and origin of the energy used.

To address these limitations, the emergy approach has gained ground as a more comprehensive alternative. Proposed by Odum (1996), this framework introduces emergy as the total amount of solar equivalent energy (seJ) required to produce a good or service. It includes both direct energy (solar, hydraulic, thermal) and indirect energy (accumulated resources such as soil, water, or human labor), enabling a more complete evaluation of the ecological impact of production systems.

In livestock systems, this perspective has been applied in various global contexts: In Argentina, Rotolo *et al.* (2007) conducted an emergy analysis of a beef cattle production system in the Pampas region, finding that 86% of the emergy used came from renewable resources. This system had an EYR above 8 and an ELR of 0.35, indicating high efficiency and low environmental impact.

In the Brazilian Pantanal, Aguiar *et al.* (2010) reported that traditional extensive cattle systems used 98% renewable resources, achieving ESI values above 20-placing these systems as benchmarks of sustainability from an emergy perspective. In France, Vigne *et al.* (2013) compared intensive and extensive dairy systems,

noting that the former heavily relied on external inputs, with an EYR < 2 and ELR > 3.5, which significantly reduced their environmental sustainability.

In Brazil and Colombia, Molina *et al.* (2025) and Molina Benavides *et al.* (2025) reported that semi-intensive and intensive systems could improve their ESI by integrating agroecological practices such as pasture rotation, silvopastoral systems, and the use of organic fertilizers.

A noteworthy element in energy modeling is the use of tools like EmSim, a platform based on Odum (1996), energy systems language, which translates energy flow diagrams into mathematical equations. Although still underutilized in Latin America, this tool represents an opportunity to simulate complex interactions among system components and to evaluate future scenarios based on fundamental energy indicators.

Multivariate Analysis and Simulation for System Characterization

From the perspective of the variability found in bovine production systems-determined by factors such as climate, altitude, scale, breeds, infrastructure, and access to technology-multivariate statistical models are powerful tools for the typification and classification of productive systems. These methods not only reduce the complexity of large datasets but also enable the grouping of production units with similar characteristics, supporting decision-making and the identification of patterns.

In Colombia, Cruz *et al.* (2023) demonstrated the application of principal component analysis (PCA) and hierarchical clustering on 68 livestock farms in Cundinamarca, resulting in the classification of systems into three groups according to their degree of technification, type of production (beef, dairy, dual-purpose), and climatic variables. This segment made it possible to identify the strengths and limitations of each group, linking them to their energy and environmental performance.

Complementarily, Durana *et al.* (2023) conducted a participatory simulation exercise in the Colombian Andean highlands to redesign small-scale dairy systems. In this case, quantitative methods were integrated with qualitative approaches, using productive simulations and sustainability assessment tools (emergy, water footprint, carbon footprint, and community capital indicators). This type of multidimensional work enhances the social validation of the proposed models and improves their adoption by local stakeholders.

Internationally, there has been a growing trend toward the use of hybrid approaches that combine energy simulation, mathematical modeling, multivariate analysis, and optimization tools. Tedeschi (2004) proposed a general architecture for validating livestock simulation models, emphasizing the importance of establishing criteria for robustness, accuracy, and contextual applicability.

Additionally, techniques such as neural networks, genetic algorithms, and Bayesian analysis are beginning to be used to simulate energy efficiency and predict productive responses to environmental or management changes.

Evaluation of Existing Models in Energetic and Emery-Based Livestock Modeling

The use of energetic and emery-based models applied to livestock systems has significantly advanced the understanding of complex processes related to energy conversion and environmental sustainability. However, a critical analysis of the most widely used approaches reveals structural limitations, methodological shortcomings, and contextual challenges that must still be addressed to enhance their scientific value, technical applicability, and relevance across diverse territories.

Strengths of Current Models

Models used in the analysis of livestock systems enable researchers to address the complexity of these productive environments through frameworks that simplify their structure while preserving their functional logic. Tools such as Odum (1996), energy diagrams, implemented through platforms like EmSim, and multivariate statistical methods available in R, allow for the disaggregation of agroecosystems into fundamental components (inputs, internal processes, and derived products). This facilitates a clearer analysis of system interactions, critical points, and efficiency levels. One of the standout advantages of these approaches is their adaptability, as they can be applied to various types of bovine production systems (beef, dairy, or mixed) and adjusted to both high- and low-technification contexts. Classical models such as Brody or Gompertz have demonstrated considerable utility in both extensive and intensive systems, having been successfully calibrated in diverse production settings (Gómez *et al.*, 2008).

Beyond their descriptive capacity, emery modeling offers a valuable quantitative perspective across different systems, through the use of standardized indicators such as the Emery Yield Ratio (EYR), Environmental Loading Ratio (ELR), and Emery Sustainability Index (ESI). These metrics make it possible to compare bovine production systems with diverse structures, facilitating the assessment of their environmental performance and projecting their behavior under future production scenarios (Rotolo *et al.*, 2007; Vigne *et al.*, 2013). When these models are integrated with technological tools for computational simulation, the analytical possibilities are greatly expanded, as they enable exploration of system responses to change factors such as climate variation, input price fluctuations, or shifts in territorial policy frameworks. In this sense, modeling becomes a strategic tool for forecasting scenarios and guiding technical and policy decisions in an agricultural context increasingly marked by uncertainty.

Identified Limitations

Although energetic and emergy-based modeling has demonstrated significant progress in its application to livestock systems, it still faces limitations that hinder its widespread use, especially in rural contexts. One of the primary challenges is the need for detailed and reliable information on key aspects such as resource consumption, productivity levels, access to natural inputs, and associated energy costs. This requirement becomes a major barrier in areas characterized by poor data availability and low organizational capacity common condition in smallholder or peasant production units (Cruz *et al.*, 2023; Durana *et al.*, 2023). Furthermore, there is a marked lack of integration of social, cultural, or organizational variables, which are fundamental in peasant systems. While energy models adequately address the biophysical dimension of sustainability, they often overlook factors related to family dynamics, gender roles, or local governance structures (Toledo, 2003; Altieri & Nicholls, 2012).

Another significant obstacle lies in the technical complexity of certain tools. Platforms such as EmSim or modeling languages like Python require advanced knowledge in mathematics, programming, or systems analysis-skills not commonly held by field professionals, extension technicians, or livestock producers (Vigne *et al.*, 2013). This technological barrier becomes even more pronounced in the absence of parallel training processes that facilitate the adoption of such methodologies. Finally, it is important to note that many existing models have been designed and tested under experimental or simulated conditions, and their empirical validation in real-world contexts remains limited (Rotolo *et al.*, 2007). This compromises the reliability of their outputs when applied directly to complex and diverse production systems, highlighting the need to design more participatory, context-specific, and empirically grounded methodologies.

Contextual Relevance and Adaptability

One of the most critical limitations of current energetic and emergy-based models is their limited adaptability to the territorial diversity of livestock systems, particularly in Latin America. Rural contexts are characterized by ecological, social, and economic heterogeneity, which calls for more flexible, participatory modeling approaches that are sensitive to local knowledge systems. Recent studies by Durana *et al.* (2023) and Postigo *et al.* (2024), emphasize that co-creating models with local actors not only improves the relevance and applicability of these tools but also fosters community empowerment and enhances systemic sustainability. Considering this reality, a critical evaluation points to the urgency of moving toward a new paradigm of livestock modeling that blends scientific accuracy with user-friendliness, systemic thinking with territorial anchoring, computational tools with pedagogical processes, and quantitative rigor with socio-environmental understanding.

Contextual Discussion: A Model Adapted to Rural Territories in Sumapaz

In Cundinamarca (Colombia), a region marked by notable ecological and sociocultural diversity, these differences become a key factor for model design. In territories with such characteristics, emergy-based models stand out for their ability to systemically integrate ecological flows and provide sustainability assessments from a holistic perspective. Nonetheless, their practical application still faces challenges, particularly due to the need for specific, high-resolution data.

On the other hand, multivariate models and participatory simulation approaches demonstrate greater territorial adaptability, as they allow for the incorporation of biophysical, socioeconomic, and cultural variables, which in turn facilitate collaboration and integration with rural communities. These methodologies can be developed using accessible statistical tools (such as PCA or cluster analysis) and are well-suited for rapid diagnostic processes and the co-creation of solutions. This type of approach allows for the evaluation not only of energy efficiency or productive performance but also of the cultural, ecological, and territorial dynamics that make these rural livestock systems unique.

Proposed Future Directions for Energetic-Emergy-Based Modeling in Livestock Systems

Based on the review of the state of the art and a critical evaluation of existing models, there is a clear need to move toward new methodological strategies that allow for more integrative, contextual, and decision-oriented modeling of livestock systems. In this regard, five key development lines are proposed for a new generation of hybrid, territorial, and participatory energy-emergy models.

One major future direction involves combining emergy analysis with multivariate statistical tools, such as PCA, cluster analysis, and multiple regression, to build models that not only represent the system's energy flows but also enable the identification of production typologies and the projection of differentiated scenarios. This would allow for the identification of relationships between energy efficiency, productive variables, and management practices, as well as the detection and promotion of sustainability patterns among producer groups.

The design of participatory models is also proposed, developed jointly by researchers, technicians, and producers. This co-construction process not only fosters social ownership of the model but also ensures its relevance in terms of language, variables, territorial priorities, and productive culture. Accordingly, it is suggested to create open, visual, and multilingual platforms, adapted to low-connectivity contexts, with user-friendly interfaces suitable for technicians, students, and producers. A promising solution would be a simplified version of EmSim, or the development of modular templates for Excel, R, or Python with preloaded data, which could facilitate the adoption of these models at the territorial level.

CONCLUSIONS

- This critical review has revealed the potential that energetic and emergy-based modeling offers for bovine livestock systems.
- One of the most significant findings is that traditional modeling-focused exclusively on productive variables and physical balances-is insufficient to explain the complexity of livestock systems, particularly in regions with high cultural and ecological diversity, such as the Colombian Andes.
- Animal growth models, energy analyses, and emergy-based approaches offer partial tools, but they lack an integrative architecture that brings together the ecological, socioeconomic, political, and symbolic dimensions of the production system.
- Emergy modeling provides a fundamental conceptual innovation by recognizing the hidden environmental value embedded in each production unit. However, its territorial impact has been limited due to technical complexity, the need for hard-to-obtain data in rural settings, and the absence of sociopedagogical translation for communities and non-specialist actors.
- In contrast, multivariate and participatory approaches have proven to be more adaptable, yet they still require improved predictive capacity, ecological precision, and energetic grounding.
- In this regard, the future of modeling in livestock systems does not lie in refining disciplinary models, but in designing hybrid architectures that foster dialogue among forms of knowledge, scales, and languages-models that integrate calculation and culture, simulation and lived experience, mathematics and territory; models that not only measure efficiency or sustainability, but also activate rural transformation, productive sovereignty, and environmental justice.
- This calls for a rupture with the extractive logic of technocratic models, proposing instead a relational, situated, and politically aware modeling approach.
- An ambitious and necessary scientific agenda is thus proposed: to advance toward modeling livestock systems as living landscapes, as ecological, economic, and cultural networks, where every energy flow, every environmental variable, and every productive decision is embedded within a framework of holistic understanding.

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